

**Exercise 18**

Find the derivative of the function.

$$g(x) = (x^2 + 1)^3(x^2 + 2)^6$$

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**Solution**

Take the derivative using the product rule and the chain rule.

$$\begin{aligned} g'(x) &= \frac{dg}{dx} = \frac{d}{dx}[(x^2 + 1)^3(x^2 + 2)^6] \\ &= \left[ \frac{d}{dx}(x^2 + 1)^3 \right] (x^2 + 2)^6 + (x^2 + 1)^3 \left[ \frac{d}{dx}(x^2 + 2)^6 \right] \\ &= \left[ 3(x^2 + 1)^2 \cdot \frac{d}{dx}(x^2 + 1) \right] (x^2 + 2)^6 + (x^2 + 1)^3 \left[ 6(x^2 + 2)^5 \cdot \frac{d}{dx}(x^2 + 2) \right] \\ &= [3(x^2 + 1)^2 \cdot (2x)] (x^2 + 2)^6 + (x^2 + 1)^3 [6(x^2 + 2)^5 \cdot (2x)] \\ &= 6x(x^2 + 1)^2(x^2 + 2)^6 + 12x(x^2 + 1)^3(x^2 + 2)^5 \\ &= 6x(x^2 + 1)^2(x^2 + 2)^5[(x^2 + 2) + 2(x^2 + 1)] \\ &= 6x(x^2 + 1)^2(x^2 + 2)^5(3x^2 + 4) \end{aligned}$$